Module - 1

* Current :- Flow of charge.
* Intensity of current :- Rate of flow of charge. (strength)

$I = Q/t$; $A = C/s$; unit : Ampere (A).

* The flow of 1C of charge in 1s is known as 1A.

($1A = 1C/1s$)

* Electric potential :- Capacity of charge to do work.

$V = W/Q$; $V = J/C$; unit : volts (V).

* When 1C of charge posses an energy of 1J, then the body has an electric potential of 1V.
* Potential difference :- Force which causes the electric current flow in a closed circuit.

$V_{ba} = V_b - V_a$.

* Electromotive Force :- Pressure or force which causes an electric current to flow.

unit : volt (V).

* Power :- The rate of doing work.

$P = VI$; $P = V^2/R$; $P = I^2R$; unit : watts or hp or kilowatts.

1 horse power = 746 watts.

* Energy :- Capacity to do work.

unit : J or kwh.

kwh = power in kw × time in hr.

• Resistance :- Property of a substance to oppose the flow of electric current.

Unit : ohm (Ω).

* When 1A current flowing through a conductor produces a heat at the rate of 1 J/s, then a conductor is said to have 1Ω resistance.

Q1) Consider a resistance which is connected to a 12V supply having resistance 2Ω, the what will be the charge produced in Coulombs for 10 sec?

Ans) $V = 12V$; $R = 2\Omega$

$$V = IR$$
$$12 = I \times 2$$
$$I = 6A.$$

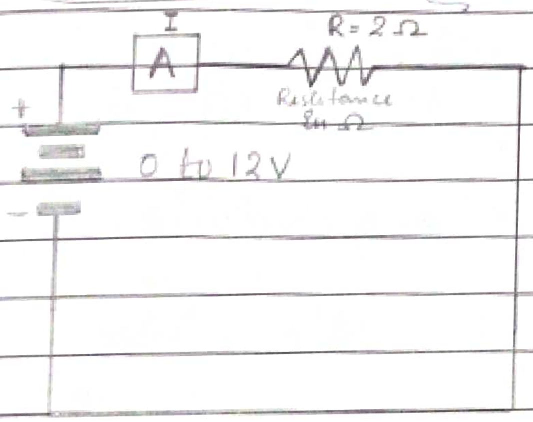
$Q = ?$; $t = 10s$; $I = 6A$

$$Q = It$$
$$= 10 \times 6$$
$$= \underline{\underline{60C}}$$

* Ohm's law : At a constant temperature, the ratio of potential difference (V) between any 2 points on a conductor to the current (I) flowing between them, is constant.

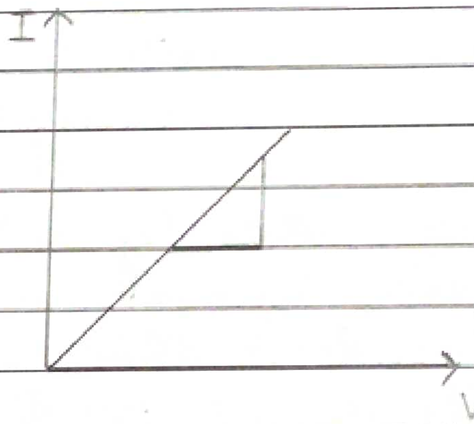
• V/I = constant
$= R$

* Volt - Ampere Characteristics (VI characteristics)



V	R	I = V/R
0	2	0
4	2	2
10	2	5
12	2	6.

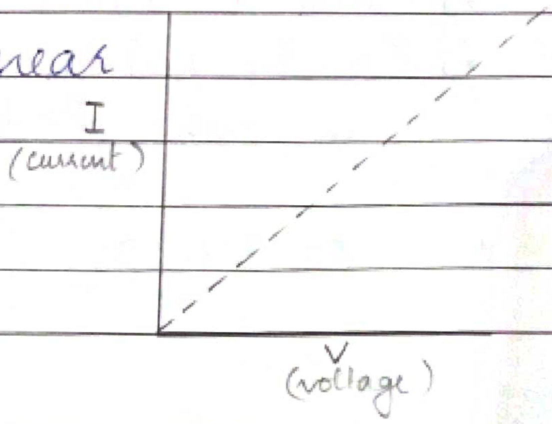
Slope = y/x
= I/V
= $1/R$



- Conductance ($1/R$)
- VI characteristics of R: Shows how much current a resistor allows.

* Linear Resistance

- Resistance doesn't vary with the flow of current through it.
- The current through it, will always be pro-portional to the voltage applied across it.
- VI charachteristic is linear
- eg: Rheostat, potentiometer



* Non Linear Resistance

- Resistance varies with the flow of current through it.
- Non Linear VI characteristics.

eg: Tungsten filament, Thermistor.

* Laws of Resistance

- The resistance offered by a conductor depends on the following factors:

- varies directly as its length, l
- varies inversely as cross sectional area, A
- depends on the nature of material
- Also depends on the temperature of the conductor.

- For const temp

$$R \propto \frac{l}{A}$$

$$R = \frac{\rho l}{A}$$

- ρ is the specific resistance or resistivity.
- when $l = 1m$, $A = 1m^2$ then $R = \rho$
- ρ is the resistance between the opposite faces of 1 m cube of that material.

* Conductor & Conductivity

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- Reciprocal of resistance (conductance)
- Conductance is defined as the potential for a substance to conduct electricity.

$$G = \frac{1}{R}$$

$$= \frac{A}{\rho l}$$

$$= \sigma \frac{A}{l}$$

- unit of σ : $ohm^{-1} m^{-1}$ or mho/m
- unit of G : mho or ℧

★ Inductor

- Inductance (L): property to oppose the change in flux.
- Conductor is twisted like a coil - basic inductor
- unit : Henry (H).

★ Current through an Inductor.

- According to Faraday's law

$$v = L \frac{di}{dt}$$

$$di = \frac{v}{L} dt$$

$$\int_0^t di = \frac{1}{L} \int_0^t v \, dt$$

$$i(t) - i(0) = \frac{1}{L}\int_{t_0}^{t} v\,dt$$

$$i(t) = \frac{1}{L}\int_{0}^{t} v\,dt.$$

- $i(0)$ is the initial current.
- Current through the inductor dependant upon the integral of the voltage of its terminals and initial current in the coil.

* Energy Stored in an Inductor

Power,

$$P = vi \qquad \text{(inductor} \Rightarrow \text{instantaneous power)}$$

$$= L\frac{di}{dt}\cdot i \qquad v = L\frac{di}{dt} \text{ (Faraday's law)}$$

Energy,

$$E = \int_0^t P\,dt$$

$$= \int_0^t L\frac{di}{dt}\cdot i \times dt.$$

$$= L\int_0^t i\,di$$

changing limits,

At $t = 0$; $i = 0$

$t = t$; $i = I$, DC current.

$$E = L\int_0^t i\,di$$

$$= \frac{1}{2}L\left[i^2\right]_0^t$$

$$= \frac{1}{2} L(I^2 - 0)$$

$$E = \frac{1}{2} L I^2$$

- If current through the inductor is constant, induced voltage = 0.
  (Inductor acts as a short circuit)

- Small change in zero time will give infinite voltage. This is practically impossible. So impulsive change in inductor current is not possible.

- A pure inductor cannot dissipate energy. Hence it is known as non-dissipative passive element.

Q1) Show that, $P = I^2R = V^2/R$ using Ohm's law.

Ans)
$$V = IR \quad - (1)$$
$$P = VI \quad - (2)$$

~~From~~ (1) Sub (1) in (2)

$$P = (IR)I$$
$$= I^2R \quad - (3)$$

multiplying & dividing the 3rd eqn by R

$$P = I^2R \times \frac{R}{R}$$
$$= \frac{(IR)^2}{R} = \frac{V^2}{R}$$

Hence proved

Q2) Calculate the resistance of 100m length of a wire having a uniform cross-sectional area of 0.1 mm^2. If the wire is made of manganin having a resistivity of 50×10^{-8} Ωm. If the the wire is drawn out to three times its original length, by how many times would you expect its resistance to be increased?

Ans) $l_1 = 100m$

$A_1 = 0.1 mm^2$

$= 1 \times 10^{-7} m^2$

$\rho = 50 \times 10^{-8} \Omega m$

$$R_1 = \frac{\rho l_1}{A_1} \quad -(1)$$

$$= 50 \times 10^{-8} \times \frac{100}{1 \times 10^{-7}}$$

$$= 50 \times 10^{-8} \times 10^{9}$$

$$= 500 \Omega$$

$$R_2 = \frac{\rho l_2}{A_2} \quad -(2)$$

∴ as the length is increased by 3 times, area decreases by 3 times.

$$l_2 = 3l_1 \quad -(3)$$

$$A_2 = \frac{A_1}{3} \quad -(4)$$

∴ Sub (3) & (4) in (2)

$$R_2 = \frac{\rho \times 3l_1 \times 3}{A_1} = \frac{9 \rho l_1}{A_1}$$

According to eqn ①

$$R_2 = 9R_1$$

★ **Capacitor (C)**

- Capacitor consist of 2 conducting surfaces separated by a layer of insulating medium called dielectric
- Capacitor stores electrical energy in dielectric.

★ **Capacitance**

- Ability to store electricity
- unit Farad, F

$$C = \frac{Q}{V}$$

- One farad is amount of capacitance when 1C charge stored with 1V across the plate.

★ **Voltage Across Capacitor**

$$C = \frac{Q}{V} \quad \text{or} \quad C = \frac{q}{v}$$

$$i = C\frac{dv}{dt} \qquad \left\{ i = \frac{dq}{dt} \right\}$$

$$dv = \frac{1}{C} i\,dt$$

$$\int_0^t dv = \frac{1}{C}\int_0^t i\,dt$$

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$$v(t) - v(0) = \frac{1}{C}\int_0^t i\,dt \qquad \left\{ v(0) \text{ is consid-ered } 0 \right\}$$

$$v(t) = \frac{1}{C}\int_0^t i\,dt + v(0)$$

* Energy stored in capacitor

$$P = VI = vi$$
$$= vC\frac{dv}{dt} \qquad \left\{ i = C\frac{dv}{dt} \right\}$$

Energy,

$$W = \int_0 p\,dt$$
$$= \int_0^t vC\frac{dv}{dt} \times dt$$
$$= C\int_0^t v\,dv = C\left[\frac{v^2}{2}\right]_0^t$$

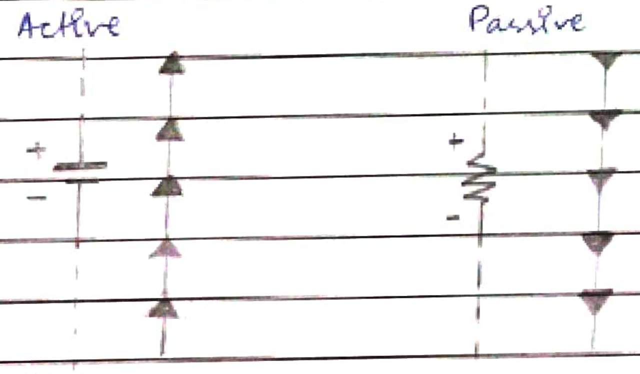
$$W = \frac{1}{2}CV^2$$

- Current in the capacitor is zero when voltage is constant.
- In a fixed capacitor, voltage cannot change abruptly
- Capacitor can store finite amount of energy even if the current through it is zero.
- Pure capacitor never dissipate energy - non-

dissipative passive element.

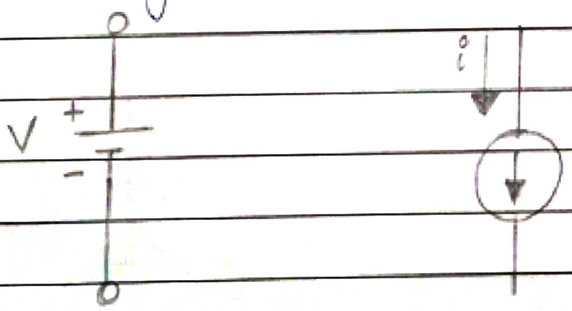
Active and passive sign convention

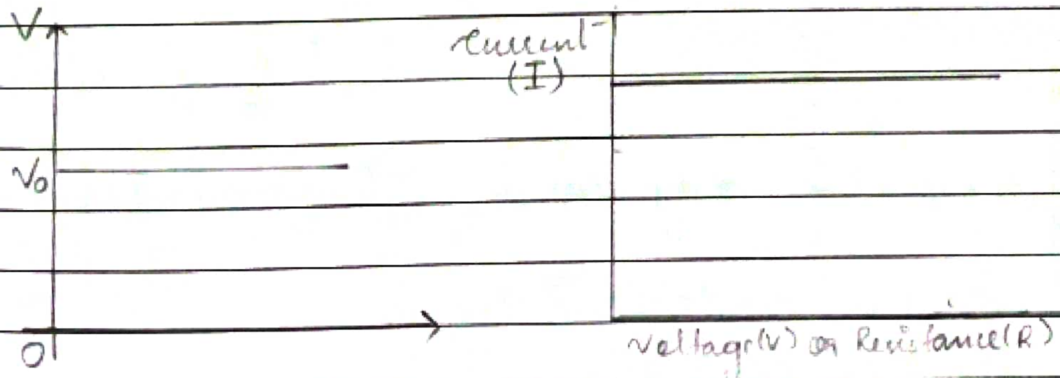
- Active sign convention is used for the devices which deliver energy to the circuit
eg: battery
- Passive sign convention is commonly used for resistors.



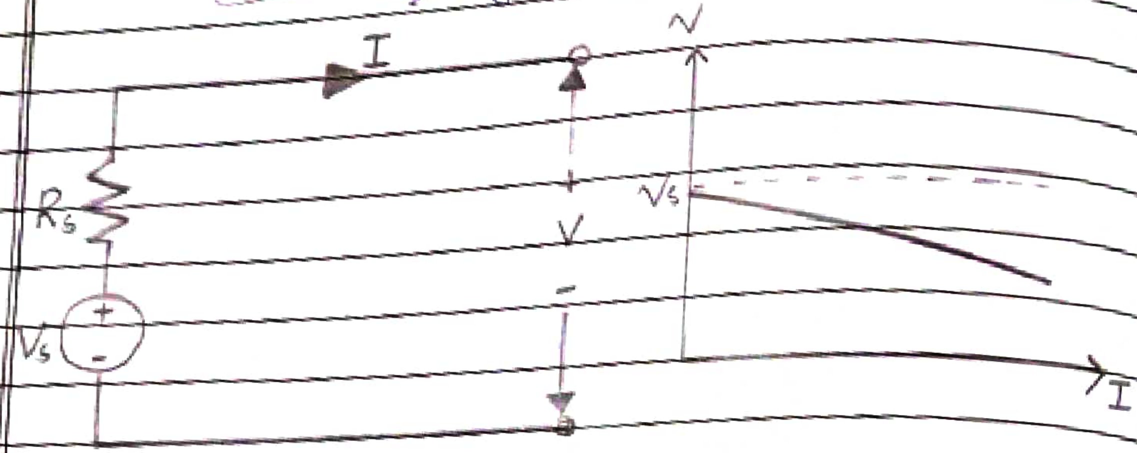
★ Energy Sources

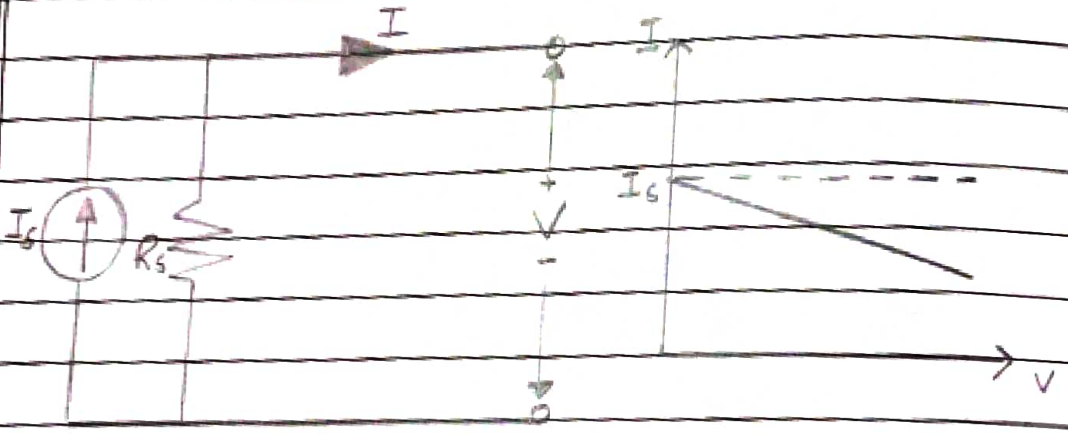
- Ideal voltage and ideal current source



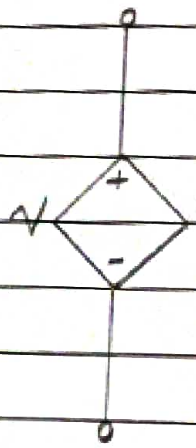


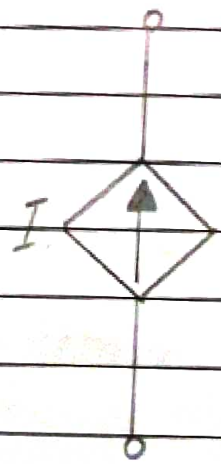
* Practical Energy Sources





* Dependent Sources





(a) Dependent voltage source (b) Dependent current source

- Voltage controlled by voltage source.
- Voltage controlled by current source.

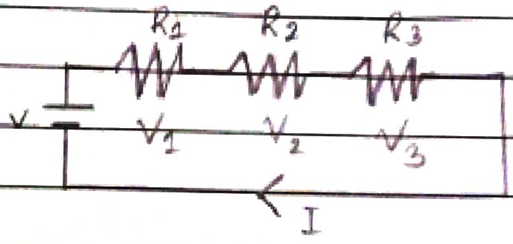
- Current controlled by voltage source.
- Current controlled by current source.

* Series Circuit

$V = IR_s$

But $V = V_1 + V_2 + V_3$

$\therefore IR_s = IR_1 + IR_2 + IR_3$



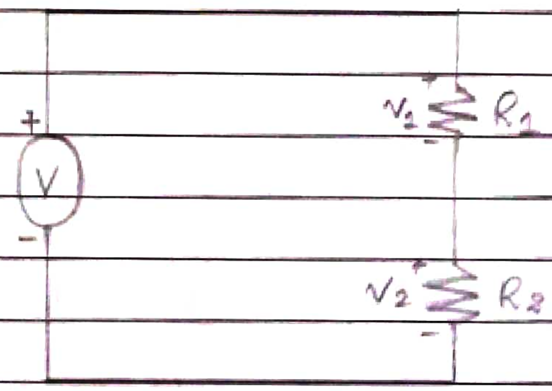
i.e. $IR_s = I(R_1 + R_2 + R_3)$

$$R_s = R_1 + R_2 + R_3$$

* Voltage Divider Rule

$R = R_1 + R_2$ — (1)

$I = \frac{V}{R}$ — (2)



$V_1 = IR_1$

$V_1 = \frac{V R_1}{R}$ from (2)

$V_1 = \frac{R_1 V}{R}$

$= \frac{R_1}{R_1 + R_2} V$ from (1)

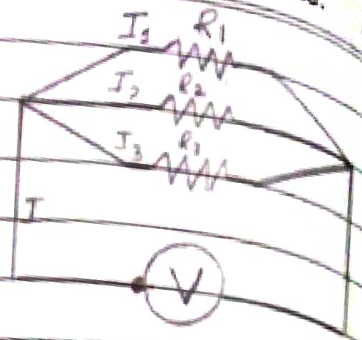
* Parallel Circuit

Total current $= I_1 + I_2 + I_3$

$I = \frac{V}{R}$

$\frac{V}{R_p} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$ $\{V_1 = V_2 = V_3 = V\}$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

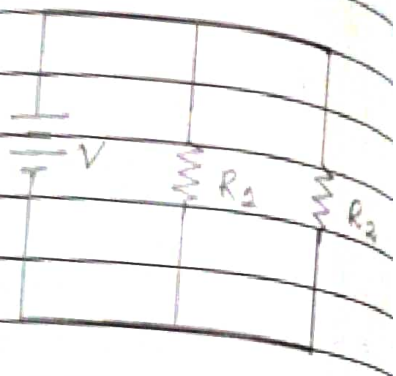


$$G = \frac{1}{R}$$

$$\therefore \quad G_p = G_1 + G_2 + G_3$$

- **Two Resistors in Parallel**

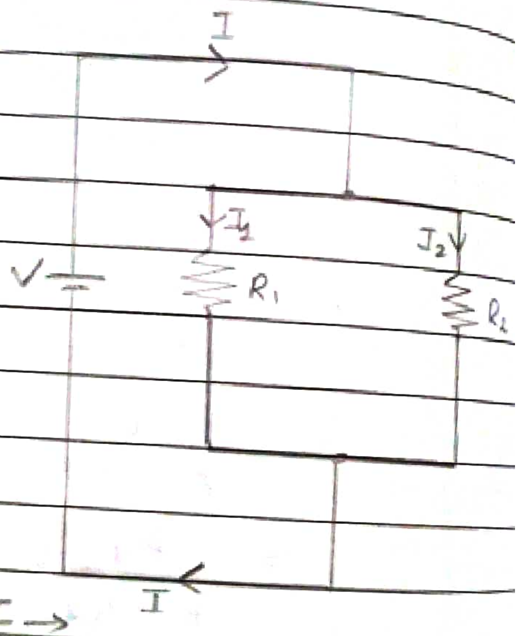
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$R = \frac{R_1 R_2}{R_1 + R_2}$$

- **Current division Rule**

$$V = IR \qquad R = \frac{R_1 R_2}{R_1 + R_2}$$

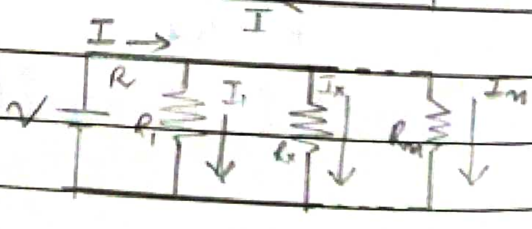


$$I_1 = \frac{V}{R_1} = \frac{IR}{R_1}$$

$$= I \times \frac{\cancel{R_1} R_2}{\cancel{R_1}(R_1+R_2)}$$

$$= \frac{R_2 I}{R_1 + R_2}$$

$$I_2 = \frac{V}{R_2}$$



$$= \frac{R_1 I}{R_1 + R_2}$$

$\therefore \quad I_x = V/R_x$

$\{V = IR\}$

$$I_x = \frac{I}{R_x} R.$$

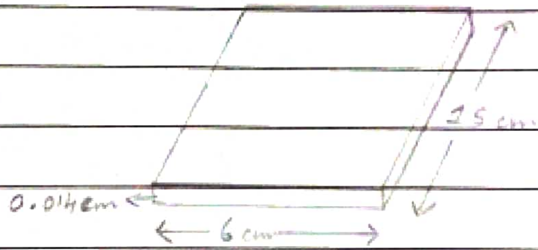
51) The resistivity of a ferric-chromium-aluminium alloy is $51\times10^{-8}\ \Omega m$. A sheet of the material is 15cm long, 6cm, wide and 0.014 cm thick. Determine resistance between

a) opposite ends and b) opposite sides.

Ans) a) $l = 15\text{ cm}$

$A = \frac{6}{100} \times \frac{0.014}{100} = 6\times14\times10^{-7}\ m^2$

$\frac{15}{100} = 0.15\ m$



$R = \frac{\rho l}{A}$

$= \frac{51\times10^{-8} \times 0.15}{\frac{6}{100} \times \frac{0.014}{100}}$

$= 51\times10^{-8} = \frac{15\times1000\cancel{0}}{6\times1\cancel{00}\times0.014}$

$= \frac{51\times\cancel{15}^{5}\times10^{-8}\times10^{3}\times10^{2}}{\cancel{6}_{2}\times14}$

$= 9.1\times10^{-3}\ \Omega$

b) $l =$

$A = \frac{15\times6}{10000}$

$= 15\times6\times10^{-4}\ m^2$

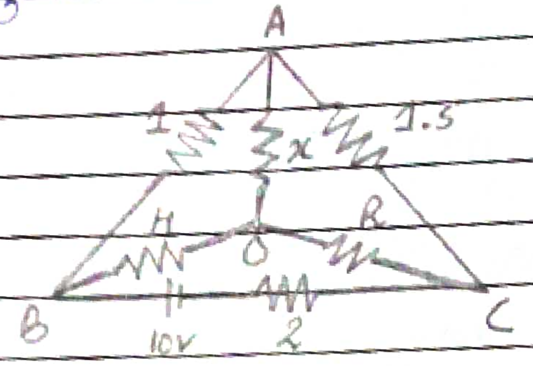
$R = \frac{\rho l}{A}$

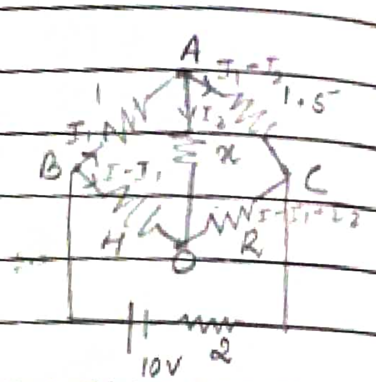
$= 51\times10^{-8} \times \frac{0.014}{15\times6\times10^{-4}}$

$= 51\times10^{-8} \times \frac{14\times10^{4}}{15\times6\times1000}$

$= \frac{51\times14}{15\times6} \times \frac{10^{-8}\times10^{4}}{1000} = 7.9\times10^{-9}\ \Omega = 79.3\times10^{-10}\ \Omega$

Q2) Determine the value of R and current through it, if current through branch AO is zero.





Ans)

$R_1 = 1\Omega$

$R_2 = 1.5\Omega$

$R_3 = 4\Omega$

$R_4 = R = ?$

$V = 10V$

$R = 2\Omega$

$I_3 = 0$

$I - I_1 = 5 - I_1$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{1}{1.5} = \frac{4}{R_4}$$

$$R_4 = 4 \times 1.5$$
$$= \underline{\underline{6\Omega}}$$

$R_{BC} = ?$

$$\frac{1}{R_{BC}} = \frac{1}{2.5} + \frac{1}{10}$$

$$= \frac{10 + 2.5}{25}$$

$$R_{BC} = \frac{25}{12.5} = \frac{250}{125} = \underline{2\Omega}$$

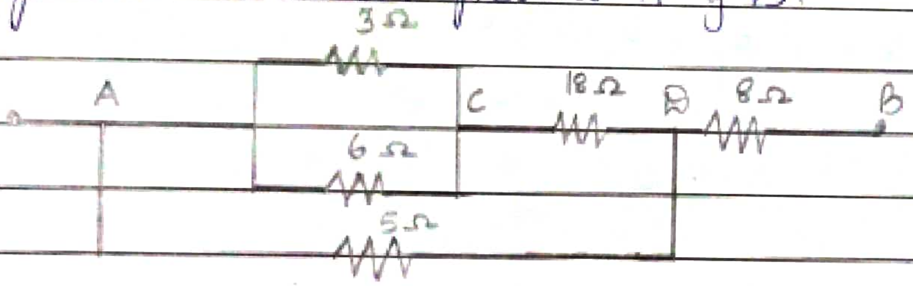
total resistance = 2 + 2 = 4 Ω

total current = $\frac{V}{R_{total}}$

$= \frac{10}{4} = \underline{\underline{2.5 A}}$

$I_R = \frac{2.5 \times 2.5}{12.5} = \underline{\underline{0.5 A}}$

Q3) Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D of 60V is applied between points A & B.



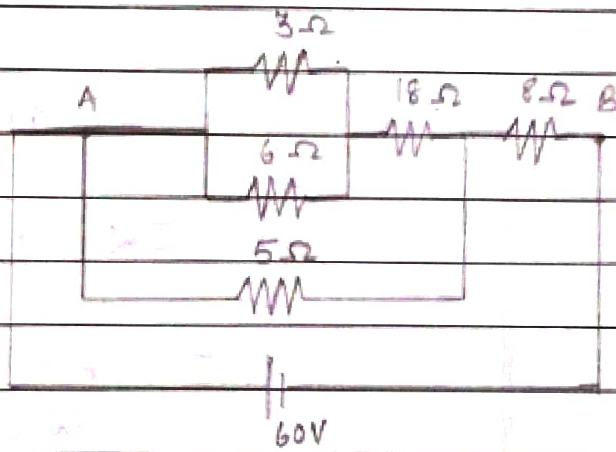
Ans) $R_1 = 3 \Omega$

$R_2 = 6 \Omega$

$R_3 = 18 \Omega$

$R_4 = 5 \Omega$

$R_5 = 8 \Omega$



$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{1}{3} + \frac{1}{6}$

$= \frac{2}{6} + \frac{1}{6} = \frac{\cancel{3}\,1}{\cancel{6}\,2}$

$R_p = 2 \Omega$

$R_S = R_P + R_3$

$= 2 + 18 = 20 \Omega$

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$$\frac{1}{R_p} = \frac{1}{R_3} + \frac{1}{R_4}$$

$$= \frac{1}{20} + \frac{1}{5}$$

$$= \frac{1}{20} + \frac{4}{20}$$

$$= \frac{5}{20}$$

$R_p = 4\,\Omega$

$R_S = R_p + R_5$
$= 4 + 8$
$= 12\,\Omega$

total effective resistance $= 12\,\Omega$

Voltage across AB is 60V

$V = IR$
$60V = I \times 12$
$I = \frac{60}{12} = 5A$

current across $5\,\Omega$ resistor

$$I_x = I \times \frac{R_2}{(R_1 + R_2)}$$

$$= \frac{5 \times 20}{25}$$

$$= 4A$$

Current across $20\,\Omega$ resistors (R_1, R_2 & R_3)

$$I_y = \frac{5\times 5}{25} \qquad \left\{ I_x = \frac{I \times R}{R_x} \right\}$$

$$= \underline{\underline{1A}}$$

∴ voltage ~~acros~~ across ~~3 & 6~~ 3Ω & 6Ω resistors

$$V = IR$$

$$V = 1A \times 2\Omega \qquad (R_p = 2\Omega)$$

$$= \underline{\underline{2V}}$$

∴ voltage across 5Ω resistor.

$$V = IR$$

$$V = 4 \times 5$$

$$= \underline{\underline{20V}}$$

∴ voltage across 18Ω resistor

$$V = IR$$

$$= 1 \times 18$$

$$= \underline{\underline{18V}}$$

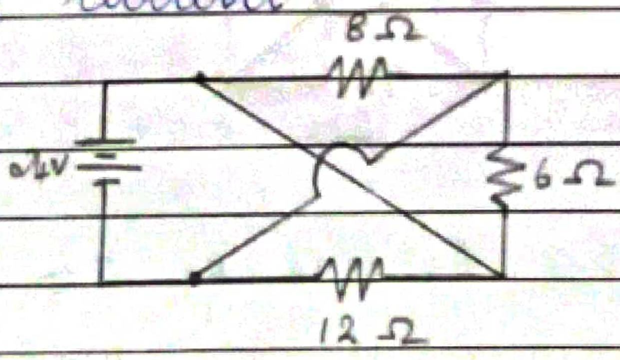
∴ voltage across 8Ω resistor

$$V = IR \qquad \{I = 4 + 1 = 5A\}$$

$$= 5 \times 8$$

$$= \underline{\underline{40V}}$$

Q4) Compute total circuit resistance and Battery current



Ans) $R_1 = 12\Omega$, $R_2 = 6\Omega$, $R_3 = 8\Omega$; $V = 24V$.

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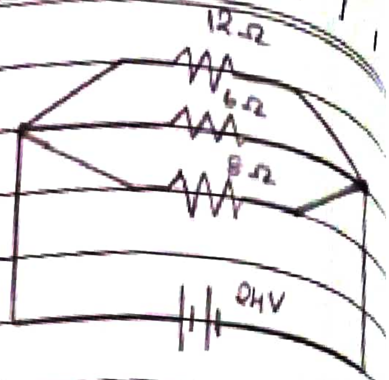


$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{12} + \frac{1}{6} + \frac{1}{8}$$

$$= \frac{2+4+3}{24}$$

$$= \frac{9}{24}$$

$$R_p = \frac{24}{9} = \underline{\frac{8}{3}}\ \Omega$$

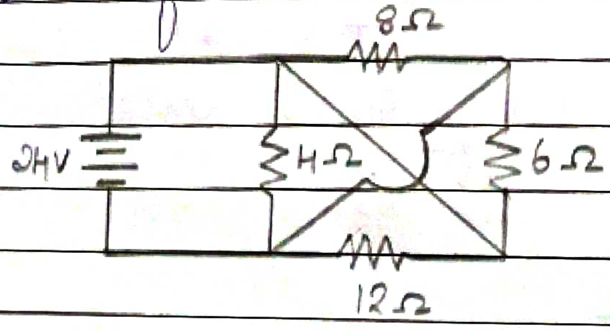
$$V = IR$$

$$24 = I \times \frac{8}{3}$$

$$\cancel{24 \times \frac{8}{8} = I} \qquad I = \cancel{24}^{3} \times \frac{3}{\cancel{8}}$$

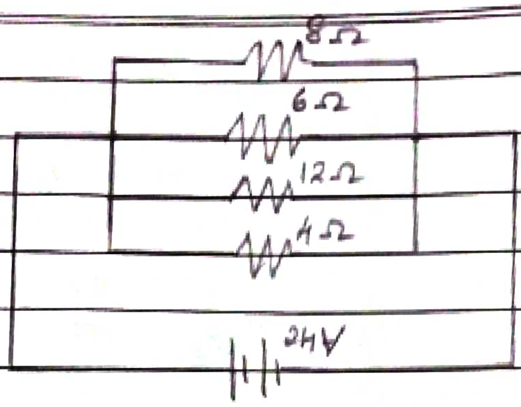
$$\cancel{I = 64\,A} \qquad = \underline{\underline{9A}}$$

Q5) Calculate battery current and equivalent resistance of the network.



Ans) $R_1 = 8\,\Omega$, $R_2 = 6\,\Omega$, $R_3 = 12\,\Omega$, $R_4 = 4\,\Omega$, $V = 24V$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$



$$\frac{1}{R_p} = \frac{1}{8} + \frac{1}{6} + \frac{1}{12} + \frac{1}{4}$$

$$= \frac{3+4+2+6}{24}$$

$$= \frac{15}{24}$$

$$R_p = \frac{\cancel{24}^{8}}{\cancel{15}^{5}} = \frac{8}{5}\ \Omega$$

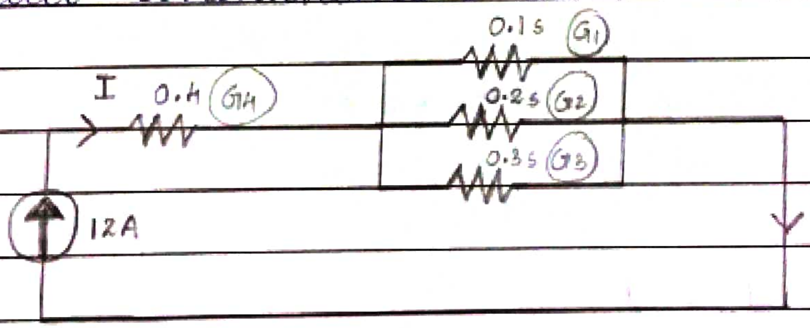
$$V = IR$$

$$24 = I \times \frac{8}{5}$$

$$I = \frac{\cancel{24}^{3} \times 5}{\cancel{8}}$$

$$= \underline{\underline{15A}}$$

96) Calculate the values of different currents for the circuit shown in the figure, what is the total circuit conductance? and resistance?



Ans) $R = \frac{1}{G}$

$$G_p = G_1 + G_2 + G_3$$

$$= 0.1 + 0.2 + 0.3$$

$$= \underline{\underline{0.6S}}$$

$$\frac{1}{G_S} = \frac{1}{G_P} + \frac{1}{G_4}$$

$$= \frac{1}{0.6} + \frac{1}{0.4}$$

$$= \frac{4+6}{24}$$

$$\frac{1}{G_S} = \frac{10}{24}$$

$$G_S = \frac{24}{10} = \underline{\underline{2.4\ S}}$$

$$R = \frac{10^{5}}{\cancel{24}\,12} = \underline{\underline{\frac{5}{12}\ \Omega}}$$

$$I_x = \frac{I \times G_x}{G}$$

$$I_1 = \frac{I \times G_1}{G}$$

$$\cancel{= \frac{12 \times 0.1 \times 10^{5}}{24\,2}}$$

$$= 12 \times \frac{0.1}{0.6}$$

$$= \frac{12}{6} = \underline{\underline{2A}}$$

$$I_2 = \frac{I \times G_2}{G}$$

$$= 12 \times \frac{0.2}{0.6}$$

$$= \cancel{12}^{\,2} \times \frac{2}{\cancel{6}}$$

$$= \underline{\underline{4A}}$$

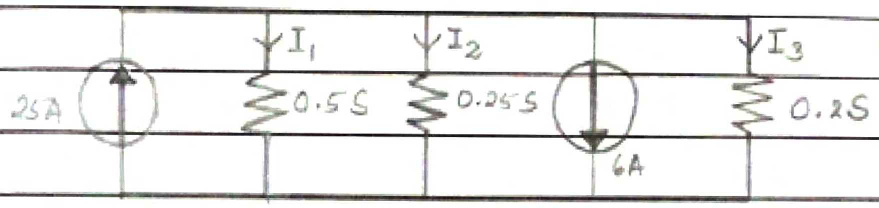
$$I_3 = \frac{I \times G_3}{G}$$

$$= \frac{12 \times 0.3}{0.6}$$

$$= \frac{\cancel{12}^{2} \times \cancel{0.}3}{\cancel{6}}$$

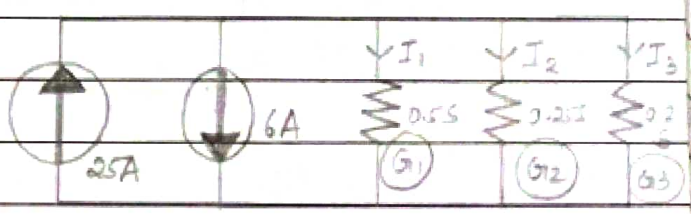
$$= \underline{\underline{6A}}$$

Q7) Compute the values of 3 branch currents for the circuit shown. What is the P.d b/w points A & B.



Ans) $I = 25 - 6$

$= \underline{\underline{19A}}$



$$I_1 = I \times \frac{G_1}{G} \quad - (1)$$

$G = ?$

$$G_p = G_1 + G_2 + G_3$$

$$= 0.5 + 0.25 + 0.2$$

$$= \underline{\underline{0.95S}}$$

$$5\overline{)95}\ 19, \quad 5, \quad 45, \quad 45, \quad 0$$

$$I_1 = 19 \times \frac{0.5}{0.95}$$

$$= \cancel{19} \times \frac{\cancel{50}^{10}}{\cancel{95}\ \cancel{19}}$$

$$= \underline{\underline{10A}}$$

$$I_2 = I \times \frac{G_2}{G}$$

$= 19 \times \frac{0.25}{0.95}$

$= 19 \times \frac{\cancel{25}^{5}}{\cancel{95}_{19}}$

$= \underline{5A}$

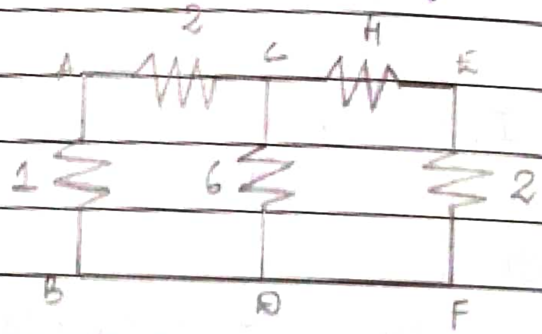
$I_3 = I \times \frac{G_3}{G}$

$= 19 \times \frac{0.2}{0.95}$

$= 19 \times \frac{\cancel{20}^{4}}{\cancel{95}_{19}}$

$= \underline{4A}$

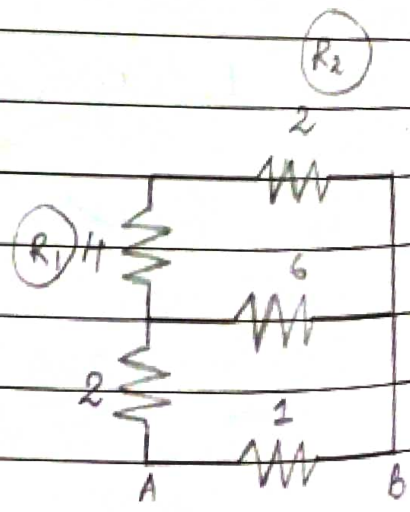
Q8) Find the equivalent resistance of the circuit shown:



(i) B/w A & B
(ii) B/w C & D
(iii) B/w E & F
(iv) B/w A & F
(v) B/w A & C.

The number represents resistances in Ω.

Ans) (i) B/w A & B.



$R_s = R_1 + R_2$

$= 4 + 2$

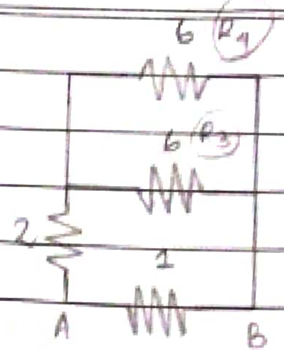
$= \underline{6\,\Omega}$

$1/R_p = 1/R_3 + 1/R_4$

$= 1/6 + 1/6$

$= 2/6$

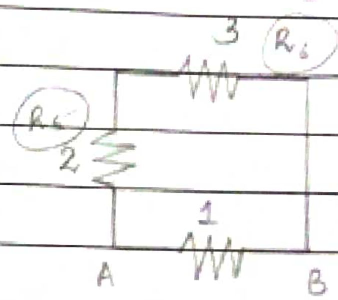
$R_p = 3\,\Omega$



$R_s = R_5 + R_6$

$= 2 + 3$

$= 5\,\Omega$



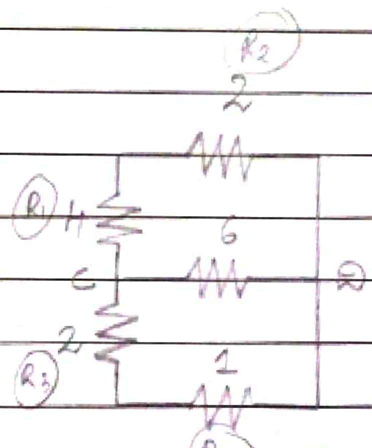
$1/R_p = 1/R_s + 1/1$

$= \frac{1}{5} + \frac{1}{1}$

$= \frac{6}{5}$

$R_p = 5/6\ \Omega$

(ii) B/w C & D



$R_{s_1} = R_1 + R_2$

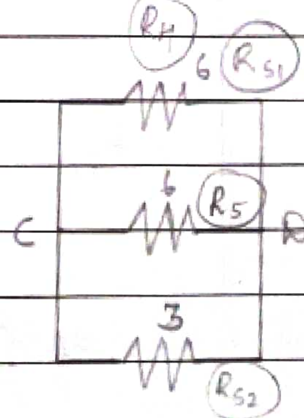
$= 4 + 2$

$= 6\,\Omega$

$R_{s_2} = R_3 + R_4$

$= 2 + 1$

$= 3\,\Omega$



$1/R_p = 1/R_{s1} + 1/R_5 + 1/R_{s2}$

$= 1/6 + 1/6 + 1/3$

$= 2/6 + 1/3$

$= 4/6$

$R_p = \frac{6}{4} = \frac{3}{2}\ \Omega$

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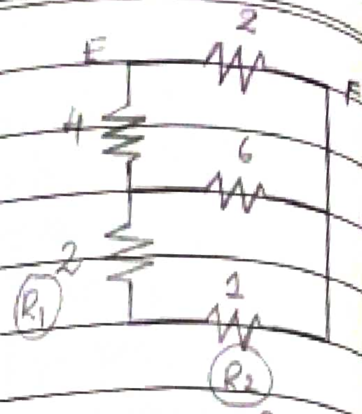
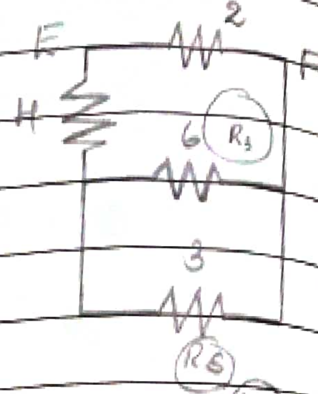


(iii) B/W E & F

$R_s = R_1 + R_2$

$= 2 + 1$

$= 3\,\Omega$



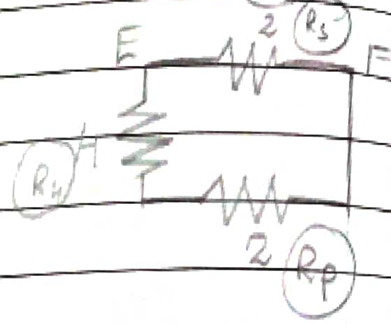
$1/R_P = 1/R_s + 1/R_3$

$= 1/3 + 1/6$

$= \frac{2+1}{6}$

$= \frac{3}{6}$

$R_P = 2\,\Omega$



$R_{S_1} = R_P + R_H$

$= 2 + 4$

$= 6\,\Omega$

$1/R_{P_1} = 1/R_{S_1} + 1/R_5$

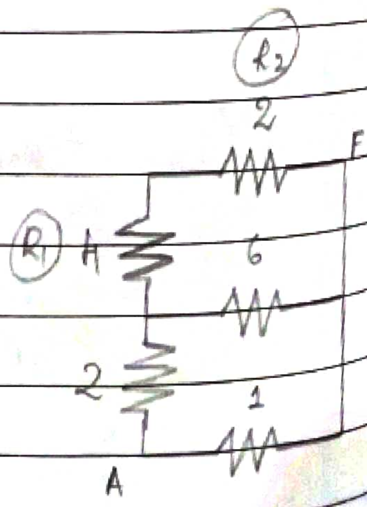
$= 1/6 + 1/2$

$= 1/6 + 3/2$

$= \frac{4}{6}$

$R_{P_1} = \frac{6}{4}\,\Omega = \frac{3}{2}\,\Omega$

(iv) B/W A & F

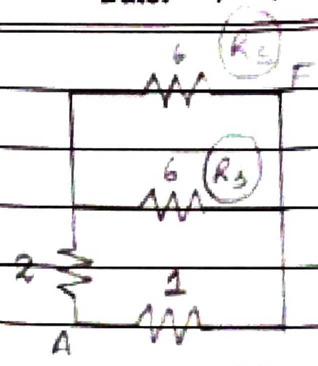


$R_s = R_1 + R_2$

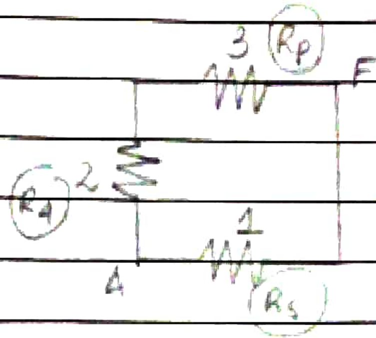
$= 4 + 2$

$= 6\,\Omega$

$$\frac{1}{R_p} = \frac{1}{R_5} + \frac{1}{R_3}$$
$$= \frac{1}{6} + \frac{1}{6}$$
$$= \frac{2}{6} = \frac{1}{3}$$
$$R_p = 3\,\Omega$$



$$R_{s_1} = R_p + R_4$$
$$= 3 + 2$$
$$= 5\,\Omega$$

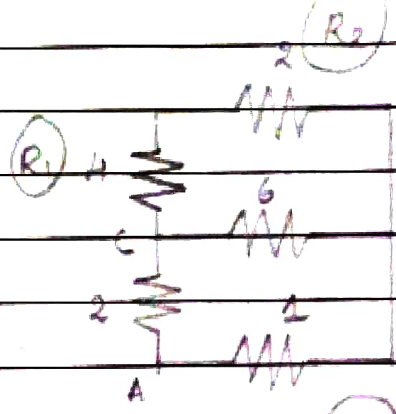


$$\frac{1}{R_{p_1}} = \frac{1}{R_{s_1}} + \frac{1}{R_5}$$
$$= \frac{1}{5} + \frac{1}{1}$$
$$= \frac{6}{5}$$

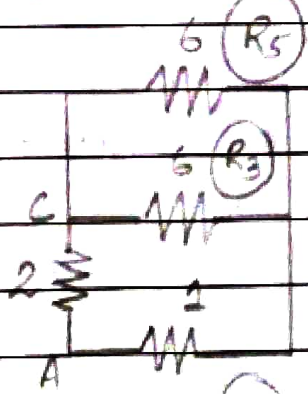
$$R_{p_1} = \frac{5}{6}\,\Omega.$$

(V) B/w A & C

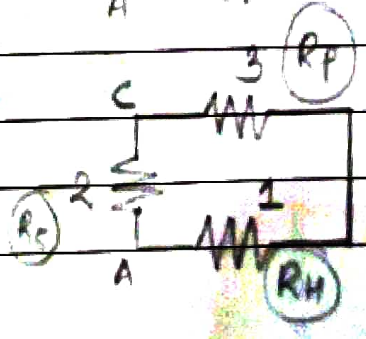
$$R = R_1 + R_2$$
$$= 4 + 2$$
$$= 6\,\Omega$$



$$\frac{1}{R_p} = \frac{1}{R_5} + \frac{1}{R_3}$$
$$= \frac{1}{6} + \frac{1}{6}$$
$$= \frac{2}{6}$$
$$R_p = 3\,\Omega$$

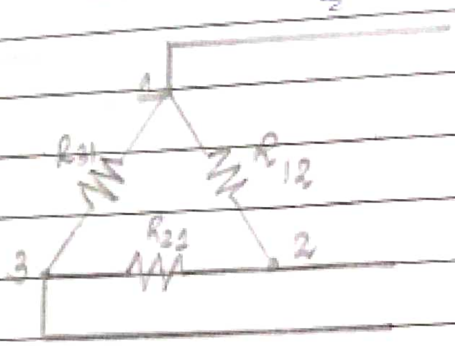


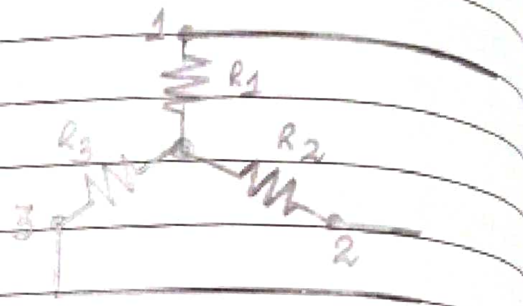
$$R_{s_1} = R_p + R_4$$
$$= 3 + 1$$
$$= 4\,\Omega$$



$$1/R_{P_1} = 1/R_{S_1} + 1/R_{S}$$
$$= 1/4 + 1/2$$
$$= 3/4$$
$$R_{P_1} = \frac{4}{3}\ \Omega$$

* Delta - Star Transformation



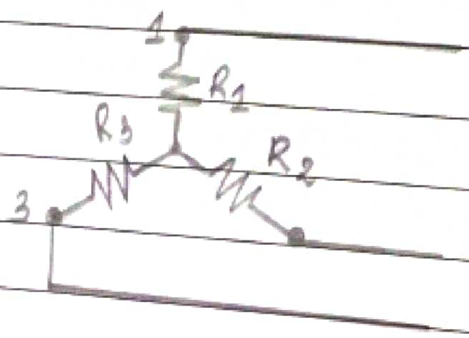


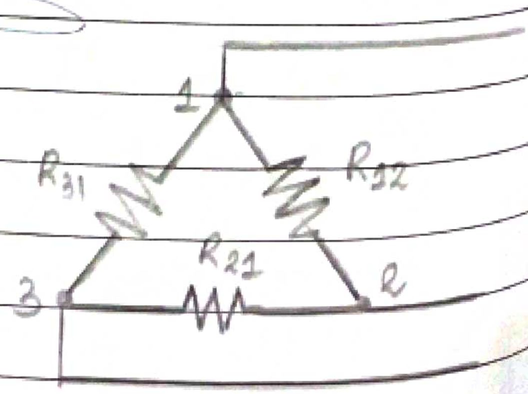
$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

* Star - Delta Transformation





$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$= \frac{R_1R_2}{R_3} + R_2 + R_1$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

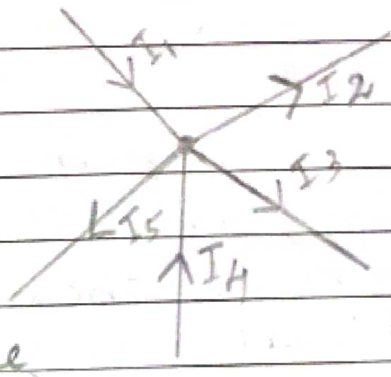
$$= R_2 + \frac{R_2R_3}{R_1} + R_3.$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$= R_1 + R_3 + \frac{R_3R_1}{R_2}.$$

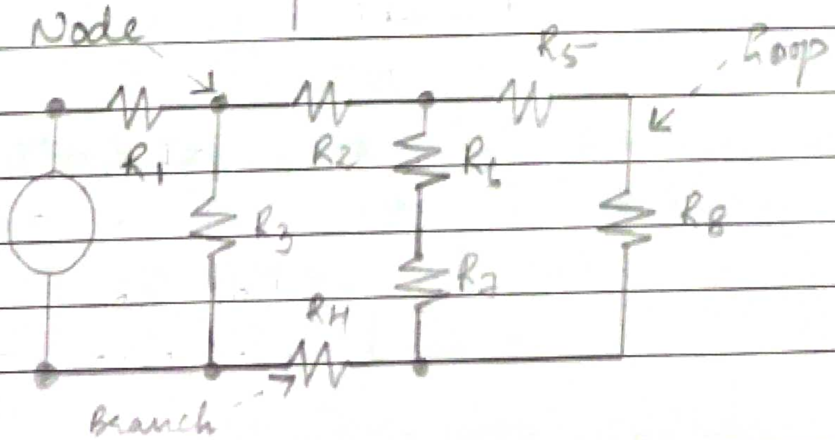
Kirchhoff's Point Law or Current Law (KCL)

- In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero.
- Incoming currents - Outgoing currents.



$$I_1 - I_2 - I_3 + I_4 - I_5 = 0$$

$$I_1 + I_4 = I_2 + I_3 + I_5$$



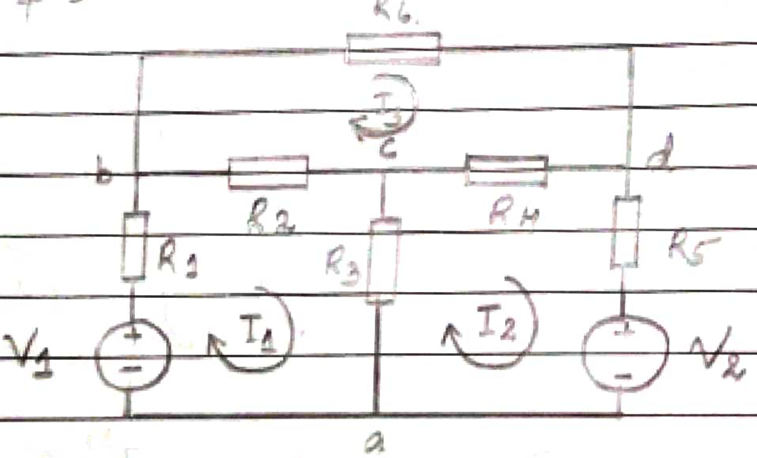
Kirchhoff's Mesh Law or Voltage Law (KVL)

- The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.fs in that path is zero.

- $\Sigma IR + \Sigma e.m.f = 0$

$V_5 + V_6 + V_7 + V_8 = 0$

Q1)



Ans)

Consider loop/mesh abca.

$-I_1R_1 - (I_1 - I_3)*R_2 - (I_1 - I_2)*R_3 + V_1 = 0$

$-I_1R_1 - I_1R_2 + I_3R_2 - I_1R_3 + R_3I_2 + V_1 = 0$

$I_1(R_1 + R_2 + R_3) - I_2R_3 - I_3R_2 = V_1$ — (1)

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Consider acda.

$$-(I_2-I_1)R_3-(I_2-I_3)(R_4)-I_2R_5-V_2=0$$
$$I_1R_3-I_2(R_3+R_H+R_5)+R_HI_3=V_2 \quad -(2)$$

Consider bdcb.

$$-R_2(I_3-I_1)-R_6(I_3)-R_4(I_3-I_2)=0$$
$$R_2I_1+R_HI_2-I_3(R_2+R_H+R_6)=0. \quad -(3).$$

$$I_1(R_1+R_2+R_3)-I_2R_3-I_3R_2=V_1 \quad -(1)$$
$$I_1R_3-I_2(R_3+R_H+R_5)+R_HI_3=V_2 \quad -(2)$$
$$I_1R_2+I_2R_H-I_3(R_2+R_4+R_6)=0 \quad -(3).$$

Matrix form:

$$\begin{bmatrix} (R_1+R_2+R_3) & -R_3 & -R_2 \\ -R_3 & (R_3+R_H+R_5) & -R_H \\ -R_2 & -R_H & (R_2+R_H+R_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix}$$

* Cramer's Rule

$$ax+by=c$$
$$dx+ey=f.$$

1) Write the two equations in the matrix form as

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$$

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2) The common determinant is given as

$$\Delta = \begin{vmatrix} a & b \\ d & e \end{vmatrix}$$
$$= ae - bd.$$

3) For finding the determinant for x, replace the coefficients of x in the original matrix by the constants so that we get determinant Δ_1 given by

$$\Delta_1 = \begin{vmatrix} c & b \\ f & e \end{vmatrix}$$
$$= ce - bf.$$

4) For finding the determinant for y, replace coefficients of y by the constants so that we get.

$$\Delta_2 = \begin{vmatrix} a & c \\ d & f \end{vmatrix}$$
$$= af - cd.$$

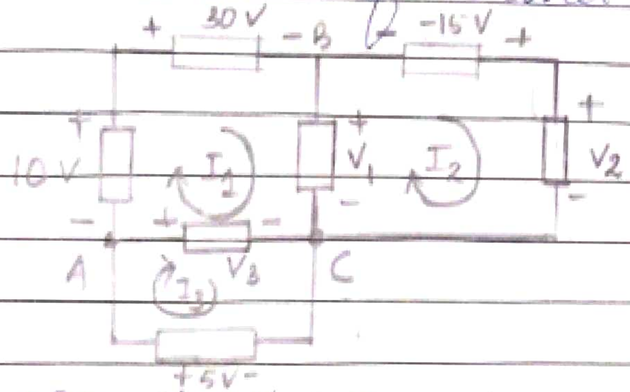
5) Apply Cramer's rule to get the value of x and y.

$$x = \frac{\Delta_1}{\Delta} = \frac{ce - bf}{ae - bd}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{af - cd}{ae - bd}$$

(IIly for 3 variables).

Q1) Applying Kirchoff's laws to different loops, find the values of V_1 and V_2.



Ans) $10 - 30 - V_1 + V_3 = 0$

$V_1 - V_3 = -20$ — (1)

$V_1 + 15 - V_2 = 0$

$V_1 - V_2 = -15$ — (2)

$-V_3 + 5 = 0$

$V_3 = 5$ — (3)

Sub (3) in (1)

$V_1 = -20 + V_3$

$= -20 + 5$

$= -15\ V$

Sub $V_1 = -15V$ in (2)

$V_1 = -15 + V_2$

$V_2 = +15 - 15$

$= 0\ V$

$\therefore V_1 = -15\ V$ & $V_2 = 0$.

Q2) Determine the current supplied by each battery in the circuit shown in the fig:

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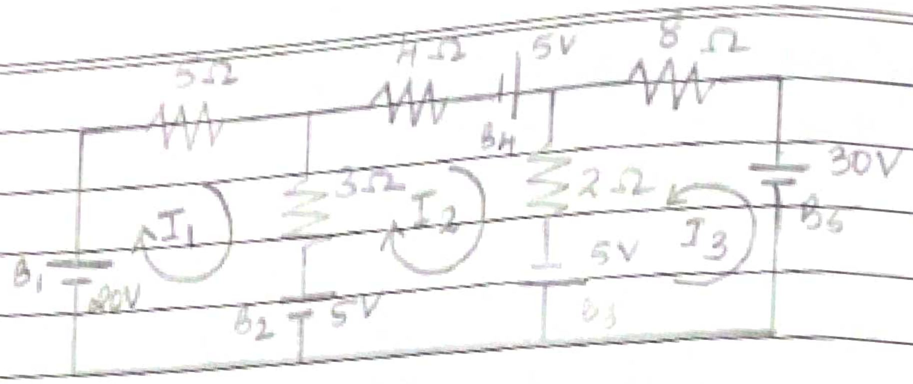

Ans) Considering the 1st loop

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$$
$$15 = 5I_1 + 3I_1 - 3I_2$$
$$8I_1 - 3I_2 = 15 \quad - (1)$$

Considering the 2nd loop

$$5 - 3(I_2 - I_1) - 4I_2 + 5 - 2(I_2 + I_3) + 5 = 0$$
$$15 - 3I_2 + 3I_1 - 4I_2 - 2I_2 - 2I_3 = 0$$
$$3I_1 - 9I_2 - 2I_3 = -15 \quad - (2)$$

Considering the 3rd loop

$$30 - 8I_3 - 2(I_2 + I_3) + 5 = 0$$
$$-8I_3 - 2I_2 - 2I_3 + 35 = 0$$
$$2I_2 + 10I_3 = 35 \quad - (3)$$

$$8I_1 - 3I_2 = 15$$
$$3I_1 - 9I_2 - 2I_3 = -15$$
$$2I_2 + 10I_3 = 35$$

$$\begin{bmatrix} 8 & -3 & 0 \\ 3 & -9 & -2 \\ 0 & 2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ -15 \\ 35 \end{bmatrix}$$

$$\Delta = 8(-90+4) - (-3)(30)$$
$$= 8 \times -86 + 90$$
$$= -688 + 90$$
$$= \underline{\underline{-598}}$$

For finding I_1

$\Delta_1 = ?$

$$\begin{bmatrix} 15 & -3 & 0 \\ -15 & -9 & -2 \\ 35 & 2 & 10 \end{bmatrix} \begin{bmatrix} \cancel{I_1} \\ \cancel{I_2} \\ \cancel{I_3} \end{bmatrix}$$

$$\Delta_1 = 15(-90+4) - (-3)(-150+70)$$
$$= -1290 - 240$$
$$= \underline{\underline{-1530}}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-1530}{-598}$$
$$= \frac{1530}{598}$$
$$= \underline{\underline{2.55\,A.}}$$

For finding I_2

$$\begin{bmatrix} 8 & 15 & 0 \\ 3 & -15 & -2 \\ 0 & 35 & 10 \end{bmatrix}$$

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$\Delta_2 = 8(-250+70) - 15(30)$

$= -640 - 450$

$= -1090$

$I_2 = \frac{\Delta_2}{\Delta}$

$= \frac{-1090}{-598}$

$= 1.82A$

For finding I_3

$$\begin{vmatrix} 8 & -3 & 15 \\ 3 & -9 & -15 \\ 0 & 2 & 35 \end{vmatrix}$$

$\Delta_3 = 8(-9(35)+30) - (-3)((35)3) + 15(6)$

$= -2280 + 315 + 90$

$= -2280 + 405$

$= -1875$

$I_3 = \frac{\Delta_3}{\Delta}$

$= \frac{-1875}{-598}$

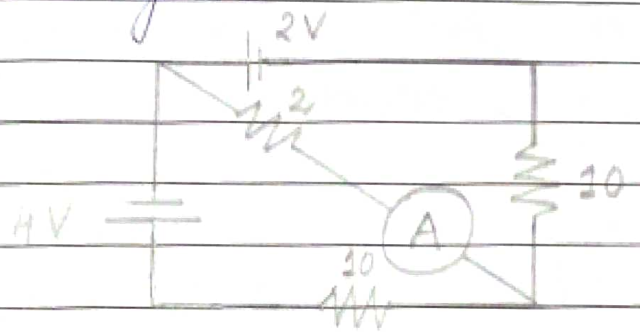
$= 3.13A$

Current flowing through $B_1 = I_1 = 2.55A$
Current flowing through $B_2 = I_1 - I_2 = 0.73A$
Current flowing through $B_3 = I_2 + I_3 = 4.95A$
Current flowing through $B_4 = I_2 = 1.82A$

Current flowing through R_5 - $I_3 = 3.13\,A$

H.W

Q1) Find the ammeter current in the fig using loop analysis:



Ans) $4 - 10I_1 - 2(I_1 - I_2) = 0$

$4 - 10I_1 - 2I_1 + 2I_2 = 0$

$-12I_1 + 2I_2 = -4$

$6I_1 - I_2 = +2 \quad -(1)$

$2 - 2(I_2 - I_1) - 10I_2 = 0$

$2 - 2I_2 + 2I_1 - 10I_2 = 0$

$2I_1 - 12I_2 = -2$

$I_1 - 6I_2 = -1 \quad -(2)$

$$\begin{bmatrix} 6 & -1 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} +2 \\ -1 \end{bmatrix}$$

$\Delta = -36 + 1$

$= -35$

For calculating I_1

$$\begin{vmatrix} 2 & -1 \\ -1 & -6 \end{vmatrix}$$

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$\Delta_1 = -12 - 1$

$= -13$

$I_1 = \frac{\Delta_1}{\Delta}$

$= \frac{-13}{-35} = \frac{13}{35}$ A

For calculating I_2

$$\begin{bmatrix} 6 & 2 \\ 1 & -1 \end{bmatrix}$$

$\Delta_2 = -6 - 2$

$= -8$

$I_2 = \frac{\Delta_2}{\Delta}$

$= \frac{-8}{-35}$

$= \frac{8}{35}$ A

$\therefore$ the current through ammeter $= I_1 - I_2$

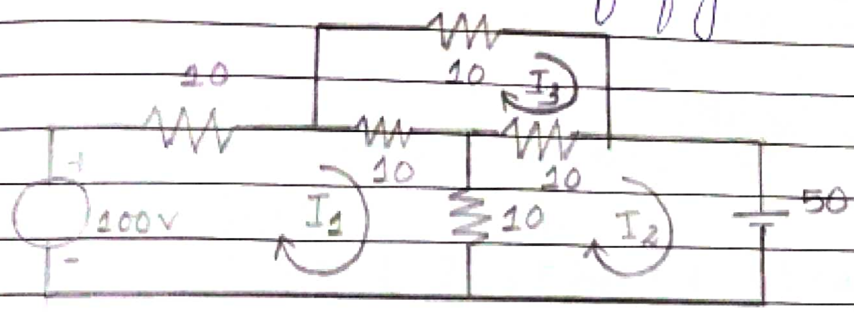
$= \frac{13}{35} - \frac{8}{35}$

$= \frac{5}{35}$

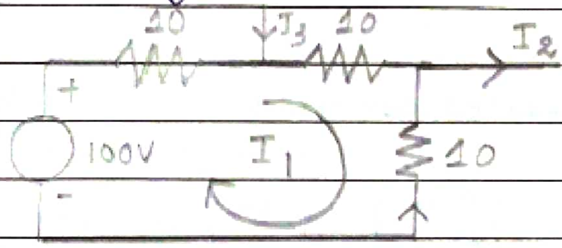
$= \frac{1}{7}$ A

2) Using ...

53) Apply loop current method to find loop currents $I_1, I_2 \& I_3$ in the circuit of fig:

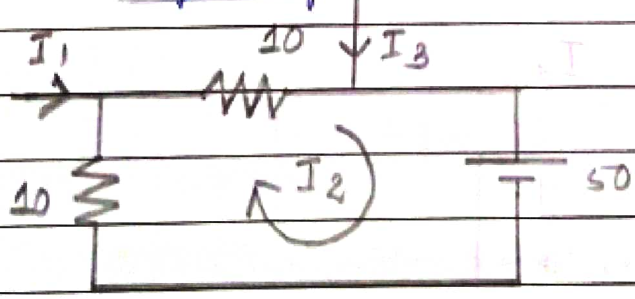


Ans) Considering loop 1



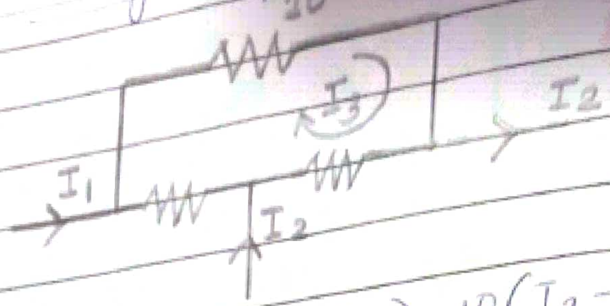
$$100 - 10(I_1) - 10(I_1 - I_3) - 10(I_1 - I_2) = 0$$
$$100 - 10\cancel{I_1} - 10\cancel{I_1} + 10I_3 - 10\cancel{I_1} + 10I_2 = 0$$
$$30I_1 - 10I_2 - 10I_3 = 100$$
$$3I_1 - I_2 - I_3 = 10 \quad \text{— (1)}$$

Considering loop 2



$$-50 - 10(I_2 - I_1) - 10(I_2 - I_3) = 0$$
$$-50 - 10I_2 + 10I_1 - 10I_2 + 10I_3 = 0$$
$$10I_1 - 20I_2 + 10I_3 = 50$$
$$I_1 - 2I_2 + I_3 = 5 \quad \text{— (2)}$$

Considering loop 3



$$-10(I_3) - 10(I_3 - I_2) - 10(I_3 - I_1) = 0$$

$$-10I_3 - 10I_3 + 10I_2 - 10I_3 + 10I_1 = 0$$

$$10I_1 + 10I_2 - 30I_3 = 0$$

$$I_1 + I_2 - 3I_3 = 0 \quad -(3)$$

Matrix representation

$$\begin{bmatrix} 3 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

$$\Delta = 3(6-1) - (-1)(-3-1) + (-1)(1+2)$$
$$= 15 - 4 - 3$$
$$= 15 - 7$$
$$= \underline{\underline{8}}$$

For finding I_1

$$\begin{vmatrix} 10 & -1 & -1 \\ 5 & -2 & 1 \\ 0 & 1 & -3 \end{vmatrix}$$

$$\Delta_1 = 10(+6-1) - (-1)(-15) + (-1)(5)$$
$$= 50 - 15 - 5$$
$$= 50 - 20 = \underline{\underline{30}}$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$= \frac{30}{8}$$

$$= \underline{\underline{3.75\ A}}$$

$$\begin{array}{r} 3.75 \\ 8\overline{)30} \\ 24 \\ \hline 60 \\ 56 \\ \hline 40 \end{array}$$

For finding I_2

$$\begin{vmatrix} 3 & 10 & -1 \\ 1 & 5 & 1 \\ 1 & 0 & -3 \end{vmatrix}$$

$$\Delta_2 = 3(-15) - (10)(-3-1) + (-1)(-5)$$

$$= -45 + 40 + 5$$

$$= \underline{\underline{0}}$$

$$I_2 = \frac{\Delta_2}{\Delta}$$

$$= \frac{0}{8} = \underline{\underline{0A}}$$

For finding I_3

$$\begin{vmatrix} 3 & -1 & 10 \\ 1 & -2 & 5 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\Delta_3 = 3(-5) - (-1)(-5) + 10(1+3)$$

$$= -15 - 5 + 40$$

$$= \underline{\underline{20}}$$

$$\begin{array}{r} 1.25 \\ 8\overline{)20} \\ 8 \\ \hline 20 \\ 16 \\ \hline 4 \end{array}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{20}{8} = \underline{\underline{1.25\ A}}$$

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$\therefore \quad I_1 = 3.25\,A$

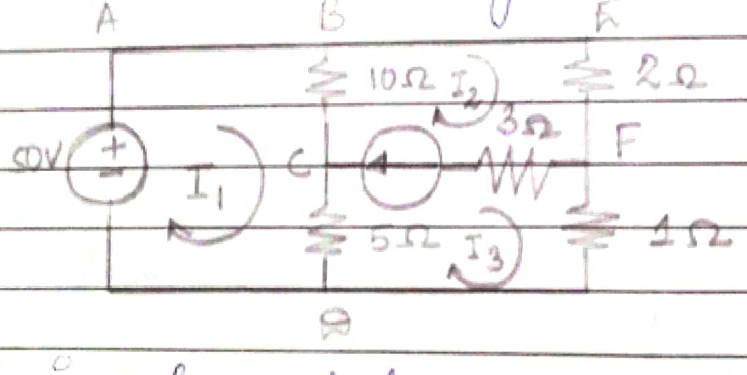
$I_2 = 0\,A$

$I_3 = 1.25\,A$

Mesh Analysis

- Identify the loops / meshes.
- Assign a current variable to each mesh/loop. using a consistent direction (clockwise or anticlockwise).
- Write Kirchhoff's Voltage Law equations around each mesh.
- Solve the resulting system of equations for all mesh currents.
- Solve for other elements currents and voltages you want using Ohm's law.

Q4) Determine the current in the 5 Ω resistor in the network given in fig:



Ans) Considering loop abda

$$50 - 10(I_1 - I_2) - 5(I_1 - I_3) = 0$$

$$50 - 10I_1 + 10I_2 - 5I_1 + 5I_3 = 0$$

$$10I_1 + 5I_1 - 10I_2 - 5I_3 = 50$$

$$15I_1 - 10I_2 - 5I_3 = 50$$

$$3I_1 - 2I_2 - I_3 = 10 \quad -(1)$$

Considering loop befdcb.

$$\cancel{-2(I_2) - 3(I_2 - I_3) - 10(I_2 - I_1) = 0}$$

$$-2(I_2) - 1(I_2) - 5(I_3 - I_1) - 10(I_2 - I_1) = 0$$

$$-2I_2 - I_3 - 5I_3 + 5I_1 - 10I_2 + 10I_1 = 0$$

$$15I_1 - 12I_2 - 6I_3 = 0$$

$$5I_1 - 4I_2 - 2I_3 = 0 \quad -(2)$$

$$I_2 - I_3 = 2A. \quad -(3)$$

$$\begin{bmatrix} 3 & -2 & -1 \\ 5 & -4 & -2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{aligned} \Delta &= 3(4+2) - (-2)(-5) + (-1)(5) \\ &= 18 - 10 - 5 \\ &= 18 - 15 \\ &= 3 \end{aligned}$$

Considering I_1

$$\begin{bmatrix} 10 & -2 & -1 \\ 0 & -4 & -2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{aligned} \Delta_1 &= 10(4+2) - (-2)(4) + (-1)(+8) \\ &= 60 + 8 - 8 \\ &= 60 \end{aligned}$$

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$$I_1 = \frac{\Delta_1}{\Delta}$$

$$= \frac{60}{3}$$

$$= 20A$$

Considering I_2

$$\begin{vmatrix} 3 & 10 & -1 \\ 5 & 0 & -2 \\ 0 & 2 & -1 \end{vmatrix}$$

$$\Delta_2 = 3(4) - (10)(-5) + (-1)(10)$$
$$= 12 + 50 - 10$$
$$= 12 + 40$$
$$= 52$$

$$\begin{array}{r} 2 \\ 3\overline{)52} \\ \underline{3\downarrow} \\ 22 \end{array}$$

$$I_2 = \frac{\Delta_2}{\Delta}$$

$$= \frac{52}{3} = 17.33A$$

Considering I_3

$$\begin{vmatrix} 3 & -2 & 10 \\ 5 & -4 & 0 \\ 0 & 1 & 2 \end{vmatrix}$$

$$\Delta_3 = 3(-8) - (-2)(10) + 10(5)$$
$$= -24 + 20 + 50$$
$$= 70 - 24$$
$$= 46$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{46}{3} = 15.33A$$

Current through 5Ω resistor = $I_3 - I_1$

$= 16.33 - 20$

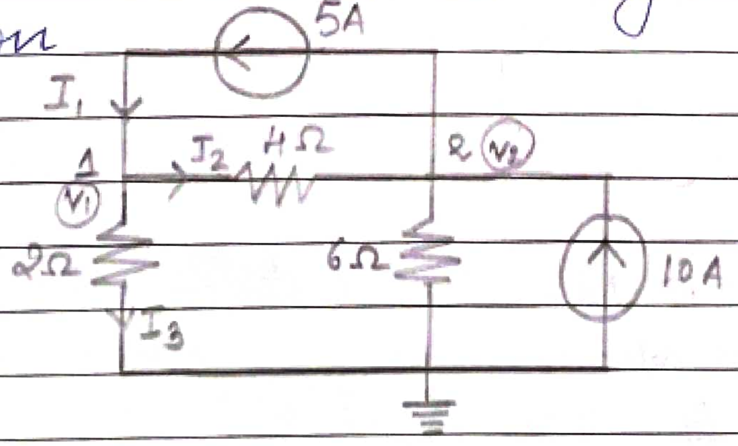
$= -4.67A$

Node Analysis

Steps to determine node voltage:

- Select a node as the reference node. Assign voltages $V_1, V_2, \ldots V_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
- Apply KCL to each of the $n-1$ nonreference nodes. Use Ohm's law to express the branch current in terms of node voltages.
- Solve the resulting simultaneous equations to obtain the unknown node voltages.

Q1) Calculate the node voltages in the circuit shown



Ans) Consider node 1

$$I_1 = I_2 + I_3$$

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$$5 = \frac{v_1 - v_2}{4} + \frac{v_1}{2}$$

$$5 = \frac{v_1 - v_2 + 2v_1}{4}$$

$$20 = 3V_1 - V_2 \quad — (1)$$

Consider node 2

$$5 - 10 + \frac{v_2 - v_1}{4} + \frac{v_2}{6} = 0$$

$$-5 + \frac{6v_2 - 6V_1 + 4V_2}{24} = 0$$

$$10V_2 - 6V_1 = 120.$$

$$5V_2 - 3V_1 = 60 \quad — (2)$$

(2)
24
×5
120

Using Cramer's rule

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$$\Delta = 15 - 3 = 12$$

Finding v_1.

$$\begin{bmatrix} 20 & -1 \\ 60 & 5 \end{bmatrix}$$

$$\Delta_1 = 100 + 60$$
$$= 160$$

$$V_1 = \frac{\Delta_1}{\Delta}$$

$$= \frac{\cancel{160}^{40}}{\cancel{12}_{3}} = \frac{40}{3} \, V$$

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Finding V_2

$$\begin{bmatrix} 3 & 20 \\ -3 & 60 \end{bmatrix}$$

$$\begin{array}{r} 180 \\ +60 \\ \hline 240 \end{array}$$

$\Delta_2 = 180 + 60$

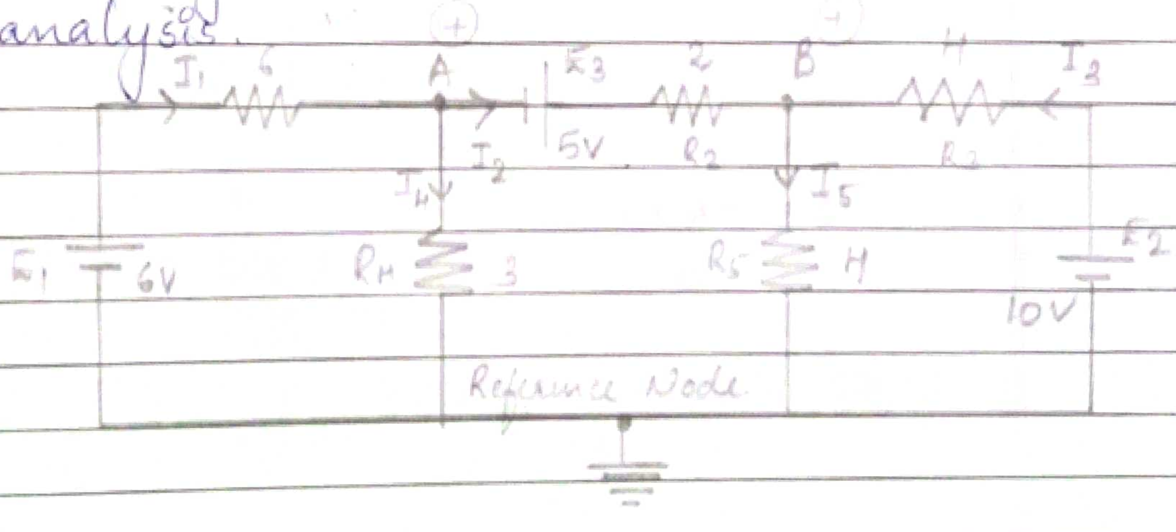
$= 240$

$V_2 = \dfrac{\Delta_2}{\Delta}$

$= \dfrac{240}{12} = \underline{\underline{20\,V}}$

Q2) Find the branch current in the circuit of fig by using

1) Nodal analysis and
2) loop analysis.



Ans) 1) Consider node 1 $(V_1 > V_2)$

$$I_1 + I_2 + I_4 = 0$$

$$\frac{V_A - 6}{6} + \frac{V_A + 5 - V_B}{2} + \frac{V_A - 0}{3} = 0$$

$$V_A - 6 + 3V_A + 15 - 3V_B + 2V_A = 0$$

$$6V_A - 3V_B + 9 = 0$$

$$2V_A - V_B + 3 = 0$$

$$V_B - 2V_A = 3 \quad - (1)$$

Considering node 2 ($V_2 > V_1$)

$$\frac{V_B - 5 - V_A}{2} + \frac{V_B}{4} + \frac{V_B - 10}{4} = 0$$

$$2V_B - 10 - 2V_A + V_B + V_B - 10 = 0$$

$$4V_B - 2V_A = 20$$

$$2V_B - V_A = 10 \quad - (2)$$

Using Cramer's rule

$$\begin{bmatrix} -2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \end{bmatrix}$$

$$\Delta = -4 + 1$$
$$= -3.$$

Considering V_A.

$$\begin{bmatrix} 3 & 1 \\ 10 & 2 \end{bmatrix}$$

$$\Delta_1 = 6 - 10$$
$$= -4$$

$$V_A = \frac{\Delta_1}{\Delta} = \frac{-4}{-3} = \frac{4}{3} \text{ V}$$

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Considering V_B

$$\begin{vmatrix} -2 & 3 \\ -1 & 10 \end{vmatrix}$$

$$\Delta_2 = -20+3$$
$$= -17.$$

$$V_B = \frac{\Delta_2}{\Delta}$$
$$= \frac{-17}{-3} = \underline{\frac{17}{3} V}$$

$I_1 = ?$

$$I_1 = \frac{-V_A + 6}{6}$$
$$= \frac{\frac{-4}{3} + 6}{6}$$
$$= \frac{-4+18}{18}$$
$$= \frac{+14}{18} A. = \frac{7}{9} A$$

$I_2 = ?$

$$I_2 = \frac{V_A - V_B + 5}{2}$$
$$= \frac{\frac{4}{3} - \frac{17}{3} + 5}{2}$$
$$= \frac{4-17+15}{6} = \frac{2}{6} = \frac{1}{3} A$$

$I_3 = ?$

$$I_3 = \frac{-V_B + 10}{4}$$

$$= \frac{\frac{-17}{3} + 10}{4}$$

$$= \frac{-17 + 30}{12}$$

$$= \frac{+13}{12} A$$

$I_4 = ?$

$$I_4 = \frac{V_A}{3}$$

$$= \frac{4}{9} A$$

$I_5 = ?$

$$I_5 = \frac{V_B}{4}$$

$$= \frac{17}{3 \times 4}$$

$$= \frac{17}{12} A$$

Using loop current method:

Consider loop 1.

$\cancel{-6I_1 - 3I_4 + 6 = 0}$ $\quad -6I_1 - 3(I_1 - I_2) + 6 = 0$

$$-6I_1 - 3I_1 + 3I_2 + 6 = 0$$
$$9I_1 - 3I_2 = 6$$
$$3I_1 - I_2 = 2 \quad - (1)$$

Considering loop 2

$$5 + (-2I_2) \mp 4(I_2 - I_3) \mp 3(I_2 - I_1) = 0$$
$$5 - 2I_2 \mp 4I_2 + 4I_3 \mp 3I_2 + 3I_1 = 0$$

$$5 + 3I_1 \mp 9I_2 + 4I_3 = 0$$
$$3I_1 - 9I_2 + 4I_3 = -5 \quad - (2)$$

Considering loop 3

$$-4I_3 - 10 \mp 4(I_3 - I_2) = 0$$
$$-4I_3 - 10 \mp 4I_3 + 4I_2 = 0$$
$$-8I_3 + 4I_2 - 10 = 0$$
$$\cancel{-4I_2 + 8I_2 = +10}$$
$$-8I_3 + 4I_2 = 10$$
$$-4I_3 + 2I_2 = 5 \quad - (3)$$

From (1)

$$3I_1 - I_2 = 2$$
$$I_2 = 3I_1 - 2 \quad - (4)$$

From (3)

$$2I_2 - 4I_3 = 5$$

$$4I_3 = 2I_2 - 5$$
$$I_3 = \frac{2I_2 - 5}{4} \quad - (5)$$

Sub (4) & (5) in (2)

$3I_1 - 9(3I_1 - 2) + \cancel{4}\left(\frac{2I_2 - 5}{\cancel{4}}\right) = -5$

$3I_1 - 27I_1 + 18 + 2I_2 - 5 = -5$

$3I_1 - 27I_1 + 18 + 2 \times (3I_1 - 2) - 5 = -5$

$3I_1 - 27I_1 + 6I_1 + 18 - 4 - 5 = -5$

~~$-18I_1 + 19 = 0$~~

~~$18I_1 = 19$~~

~~$I_1 = \frac{1}{2}$ A~~

$-18I_1 + 14 = 0$

$18I_1 = 14$

$I_1 = \frac{14}{18} = \frac{7}{9}$ A

From (4)

$I_2 = 3I_1 - 2$

$= \frac{3 \times 7}{9} - 2 = \frac{21 - 18}{9} = \frac{1}{3}$ A

From (5)

$I_3 = \frac{2I_2 - 5}{4}$

$= \frac{2 \times \frac{1}{3} - 5}{4} = \frac{2 - 15}{12} = \frac{-13}{12}$ A

$I_4 = I_1 - I_2$

$= \frac{7}{9} - \frac{1}{3} = \frac{4}{9} = \frac{2}{3}$ A

$I_5 = I_2 - I_3$

$= \frac{1}{3} + \frac{13}{12}$

$= \frac{4 + 13}{12}$

$= \frac{17}{12}$ A

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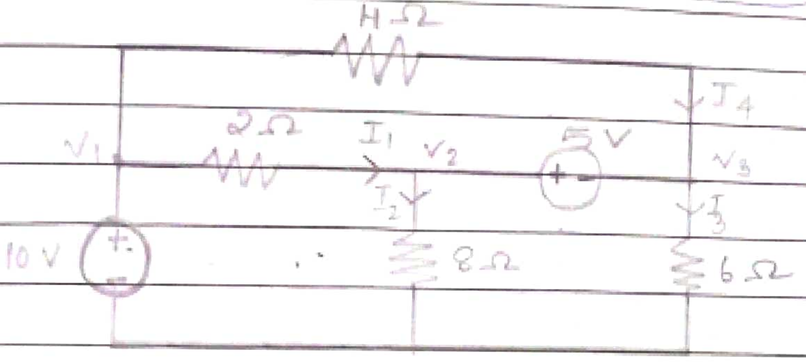
Nodal Analysis or Super node method

Q1)



Find V_1, V_2 & V_3.

Ans) Considering node 1.

$$V_1 = 10V \quad - (1)$$

Considering node 2

$$\frac{V_2}{8} + \frac{(V_2 - V_1)}{2} = 0 \quad - (2)$$

Considering node 3

$$\frac{V_3}{6} + \frac{V_3 - V_1}{4} = 0 \quad - (3)$$

Adding from (3) & (4)

$$\frac{V_2}{8} + \frac{(V_2 - V_1)}{2} + \frac{V_3}{6} + \frac{(V_3 - V_1)}{4} = 0 \quad - (4)$$

$$6V_2 + 24V_2 - 24V_1 + 8V_3 + 12V_3 - 12V_1 = 0$$

$$-36V_1 + 30V_2 + 20V_3 = 0$$

$$36V_1 - 30V_2 - 20V_3 = 0$$

$$360 = 30V_2 + 20V_3$$

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$$3V_2 + 2V_3 = 36 \quad -(5)$$

Considering the equation between V_2 & V_3

$$V_2 - V_3 = 5 \quad -(6)$$

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 36 \\ 5 \end{bmatrix}$$

$$\Delta = -3-2$$
$$= -5$$

Finding V_1

$$\begin{bmatrix} 36 & 2 \\ 5 & -1 \end{bmatrix}$$

$$\Delta_1 = -36-10$$
$$= -46$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-46}{-5}$$

$$= \frac{46}{5} V$$

Finding V_2

$$\begin{bmatrix} 3 & 36 \\ 1 & 5 \end{bmatrix}$$

$$\Delta_2 = 15 - 36$$
$$= -21$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-21}{-5} = \frac{21}{5} V$$

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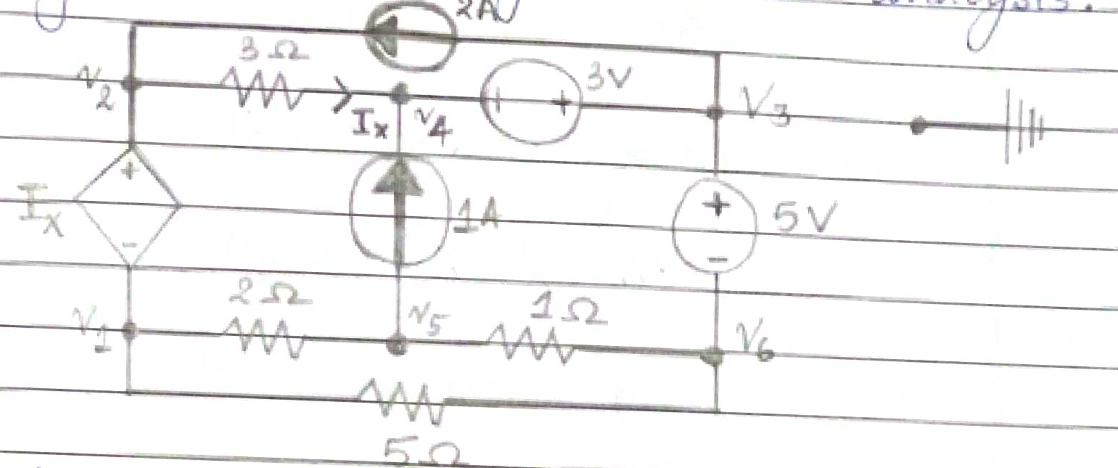
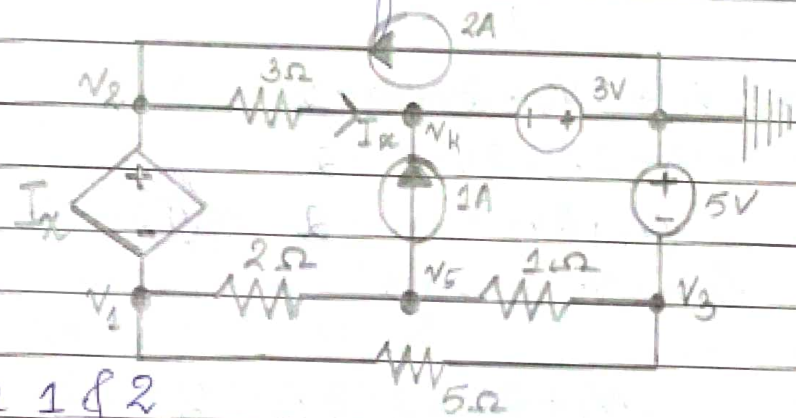
5.2) Determine the power of each source after solving the circuit by the nodal analysis.



Ans) Let V_3 be the reference node.



Node 1 & 2

We cannot apply KCL to this node directly due to the presence of voltage source in the branch shown.

∴ consider 1 & 2 as supernode

$$\frac{V_1 - V_5}{2} + \frac{V_1 - V_3}{5} + \frac{V_2 - V_4}{3} - 2 = 0$$

$$\frac{3V_1 - 3V_5 + 2V_2 - 2V_4}{6} + \frac{V_1 - V_3}{5} - 2 = 0$$

$$15V_1 - 15V_5 + 10V_2 - 10V_4 + 6V_1 - 6V_3 - 60 = 0$$

$$21V_1 + 10V_2 - 6V_3 - 10V_4 - 15V_5 = 60 \quad -(1)$$

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Considering the node 3

$$V_3 = -5V \quad -(2)$$

Considering the node 4.

$$V_4 = -3V \quad -(3)$$

Considering node 5

$$\frac{V_5 - V_1}{2} + \frac{V_5 - V_3}{1} + 1 = 0$$

$$V_5 - V_1 + 2V_5 - 2V_3 = -2$$

$$-V_1 - 2V_3 + 3V_5 = -2$$

$$V_1 + 2V_3 - 3V_5 = 2$$

Sub V_3 in eqn.

$$V_1 + 2(-5) - 3V_5 = 2$$

$$V_1 - 10 - 3V_5 = 2$$

$$V_1 - 3V_5 = 12 \quad -(4)$$

Considering the supernode.

$$I_x \Rightarrow V_2 - V_1 = 0$$

$$I_x \Rightarrow \frac{V_2 - V_4}{3} = 0.$$

$$V_2 - V_1 = \frac{V_2 - V_4}{3}$$

$$3V_2 - 3V_1 = V_2 - V_4$$

$$2V_2 - 3V_1 + V_4 = 0$$

$$3V_1 - 2V_2 - V_4 = 0$$

$$3V_1 - 2V_2 = -3 \quad -(5)$$

From eqn (1)

$$21V_1 + 10V_2 + 30 + 30 - 15V_5 = 60$$

$$21V_1 + 10V_2 - 15V_5 = 0 \quad -(6)$$

Sub (4) & (5) in (6)

$$21V_1 + \cancel{10}^{5}\left(\frac{3V_1+3}{\cancel{2}}\right) - \cancel{15}^{5}\left(\frac{V_1-12}{\cancel{3}}\right) = 0$$

$$21V_1 + 15V_1 + 15 - 5V_1 + 60 = 0$$

$$31V_1 + 75 = 0$$

$$V_1 = \frac{-75}{31}$$

$$= \underline{\underline{-2.41\,V}}$$

Sub $V_1 = -2.41$ in (5)

$$3(-2.41) - 2V_2 = -3$$

$$2V_2 = 3(-2.41) + 3$$

$$V_2 = (-7.23 + 3)/2$$

$$= \underline{\underline{-4.23\,V/2}} = \underline{\underline{-2.115\,V}}$$

Sub $V_1 = -2.41$ in (4)

$$V_1 - 3V_5 = 12$$

$$3V_5 = -2.41 - 12$$

$$V_5 = \frac{-2.41 - 12}{3}$$

$$= \frac{-14.41}{3} = \underline{\underline{-4.803\,V}}$$

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$V_1 = -2.41V$

$V_2 = -2.115V$

$V_3 = -5V$

$V_4 = -3V$

$V_5 = -4.803V$

$P_{2A} = ?$

$V_{2A} = -V_2$
$= -(-2.115) = 2.115V$

$P_{2A} = I_{2A} V_{2A}$
$= 2 \times -V_2$
$= 2 \times 2.115$
$= 4.230W$

$P_{1A} = ?$

$V_{1A} = (V_5 - V_4)$
$= (-4.803 + 3)$
$= -1.803V$

$P_{1A} = I_{1A} \times V_{1A}$
$= 1 \times -1.803$
$= -1.803W$

$P_{5V} = ?$

$I_{5V} = I_{1\Omega} + I_{3\Omega}$
$= V_3 - V_5 + \frac{V_3 - V_1}{5}$

$$= \frac{5V_3 - 5V_5 + V_3 - V_1}{5}$$

$$= \frac{6V_3 - 5V_5 - V_1}{5}$$

$$= \frac{6(-5) - 5(-4.803) \mp (-2.41)}{5}$$

$$= \frac{-30 + 24.015 + 2.41}{5}$$

$$= \frac{\cancel{-10.985}}{5} \quad \frac{-3.575\,A}{5}$$

$$P_{5V} = V_{5V} \times I_{5V}$$

$$= 5 \times \frac{-3.575}{5}$$

$$= -3.575\,W$$

$P_{3V} = ?$

$$I_{3V} = I_{3\Omega} \mp 1A$$

$$= \frac{V_4 - V_2 \mp 1}{3}$$

$$= \frac{V_4 - V_2 \mp 3}{3}$$

$$= \frac{-3 + 2.115 \mp 3}{3}$$

$$= \frac{\cancel{6} + 2.115\,A}{3}$$

$$P_{3V} = V_{3V}\, I_{3V}$$

$$= \cancel{3} \times \frac{2.115 - 6}{\cancel{3}}$$

$$= -3.885\,W$$

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$P_x = V_x I_x$

$V_x = V_2 - V_1$

$= -2.115 + 2.41$

$= -0.295\,V$

$I_x = I_{2A} + I$

$= 2 + (V_2 - V_1)$

$= 2 + (-0.295)$

$= 1.705\,A$

$P_x = V_x I_x$

$= 0.295 \times 1.705$

$= 0.502\,W$